Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 14

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A
Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Subtract $5\sqrt{3}+7\sqrt{5}$ from $3\sqrt{5}-7\sqrt{3}$.

   A. $-\left(4\sqrt{5}+12\sqrt{3}\right)$

   B. $-\left(4\sqrt{5}-12\sqrt{3}\right)$

   C. $-\left(4\sqrt{5}+12\sqrt{2}\right)$

   D. $-\left(4\sqrt{2}+12\sqrt{3}\right)$

2. If $\sqrt{3}=1.7320$ then $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}=?$

   A. 0.286
   B. 0.288
   C. 0.268
   D. 0.266

3. A conical tank is 6 m deep and its circular top has a radius 1.4 m. Find the capacity of the tank.
   A. 86.25 m^3
   B. 12.32 m^3
   C. 8.8 m^3
   D. 52.8 m^3

4. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is
   A. 5 cm
   B. 4 cm
   C. 6 cm
   D. 7 cm

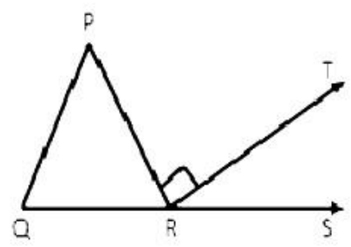
5. Find $p(-1)$, if $p(x) = 6x^3 + x^2 + 2x - 1$.
   A. –6
   B. 6
   C. –8
   D. 8

6. Find the product using appropriate identity: $(x - 5)(x + 9)$
   A. $x^2 + 4x + 45$
   B. $x^2 + 4x - 45$
   C. $x^2 - 4x - 45$
   D. $x^2 - 4x + 45$

7. If $x°$ and $(3x - 20)°$ are a pair of corresponding angles, then find x.
   A. 10
   B. 12
   C. 20
   D. 18

8. If the angles of a triangle are $x°$, $3x°$ and $5x°$, find the value of x.
   A. 10°
   B. 12°
   C. 20°
   D. 18°

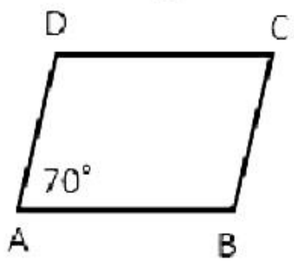
9. In the figure, side QR of a triangle PQR has been produced to S. If $\angle P:\angle Q:\angle R = 3:2:1$ and RT is perpendicular to PR, find $\angle TRS$.



A. 30°
B. 60°
C. 90°
D. 45°

10. In ΔPQR, PQ = 7.5 cm, QR = 6.2 cm and PR = 6.4 cm. Name the least angle of the triangle.
A. ∠P
B. ∠Q
C. ∠R
D. All are equal

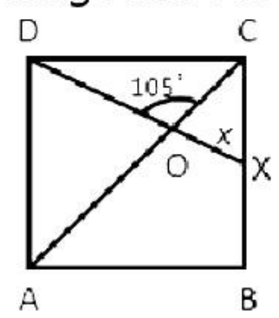
11. In the given figure, ABCD is a parallelogram in which ∠A = 70°. Calculate ∠C.



A. 30°
B. 60°
C. 70°
D. 45°

12. The measure of two angles of a quadrilateral are 60° each, and the other two angles are equal. What is the measure of each of these two angles?
A. 130°
B. 120°
C. 110°
D. 100°

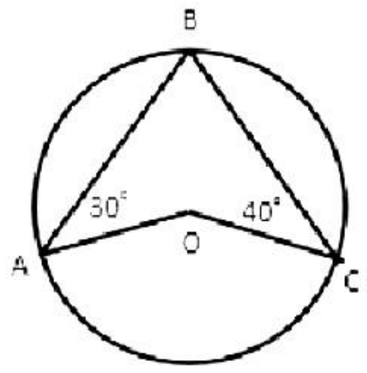
13. In the figure, ABCD is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that ∠COD = 105° and ∠OXC = x. Find the value of x.



A. 30°
B. 60°
C. 90°
D. 45°

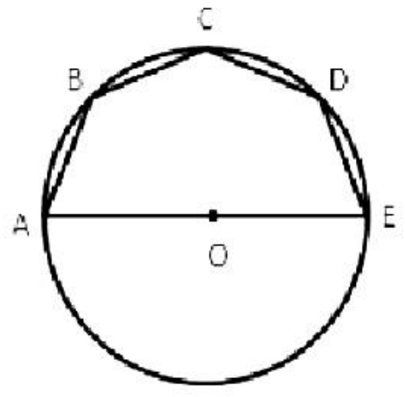
14. Write the names of the angles which are not equal when a transversal intersects two parallel lines.
A. corresponding angles
B. alternate angles
C. interior angles
D. All are equal

15. In the given figure, O is the centre of the circle, ∠OAB = 30° and ∠OCB = 40°. Find ∠AOC.



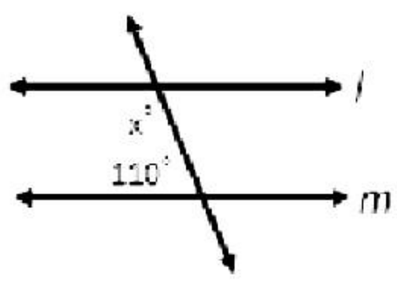
A. 130°
B. 160°
C. 150°
D. 140°

16. In the given figure, AE is the diameter of a circle. Write the numerical value of ∠ABC + ∠CDE.



A. 270°
B. 260°
C. 290°
D. 245°

17. Find the value of x from the following figure if line l || line m:

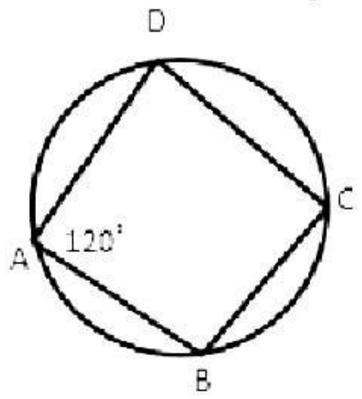


A. 30°
B. 70°
C. 90°
D. 45°

18. Write the undefined terms in geometry.
    A. points
    B. lines
    C. planes
    D. all of above

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19.**Statement A (Assertion)**: ABCD is a cyclic quadrilateral, then ∠C = 60°



Statement R (Reason): Adjacent angles of a cyclic quadrilateral are supplementary.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20.**Statement A (Assertion)**: If an angle is equal to twice its complement, then the measure of the angle is 60°.

Statement R (Reason): Sum of two complementary angles is 90°.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

Section B

Section B consists of 5 questions of 2 mark each.

21. Verify if $(x - 1)$ is a factor of the polynomial $x^4 + x^2 - 8x + 6$. [2]

22. Evaluate 203×205 without multiplying directly. [2]

23. Express $\frac{10}{7}$ in the decimal form. [2]

24. Without actually dividing, state whether the number is a terminating decimal or not: $\frac{13}{20}$ [2]

OR

Express $0.7777\ldots$ in the form of $\frac{p}{q}$

25. Without actually calculating the cubes, find the value of $\left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{-5}{4}\right)^3$ [2]

OR

Write the cubes in the expanded form: $\left(\frac{a}{2} - \frac{b}{3}\right)^3$

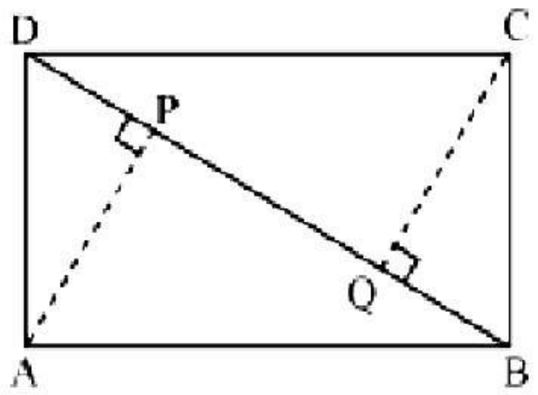
Section C

Section C consists of 6 questions of 3 marks each.

26. The sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area. [3]

27. Find the value of k, if $(2x - 3)$ is a factor of $2x^3 - 9x^2 + x + k$. [3]

28. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD.



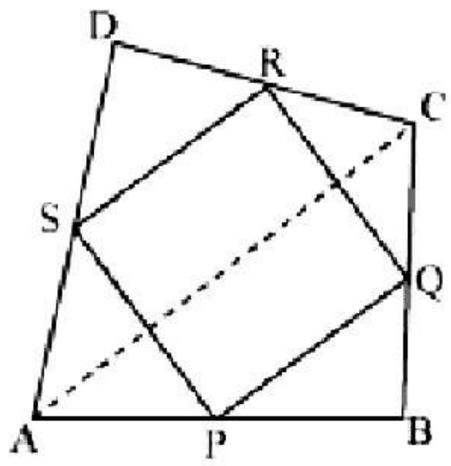
Show that

(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ [3]

OR

ABCD is a quadrilateral in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. AC is a diagonal.



Show that

(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

29. Distribution of weight (in kg) of 100 people is given below: [3]

Weight in Kg	Number of people
40–45	13
45–50	25
50–55	28
55–60	15
60–65	12
65–70	5
70–75	2

Construct a histogram for the above distribution.

30. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2. [3]

OR

A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the approximate cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$).

31. The marks obtained by Kunal in his annual examination are shown below: [3]

Subject	Hindi	English	Mathematics	Science	Social Studies
Marks	63	75	90	72	58

Draw a bar graph to represent the above data.

Section D

Section D consists of 4 questions of 5 marks each.

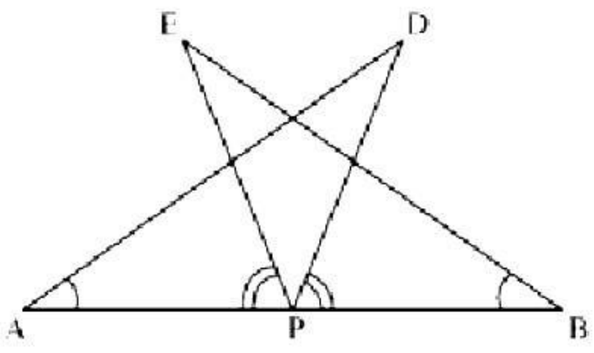
32. Suppose a, b, c are three numbers such that $a \neq b$, $b \neq c$, $c \neq a$ and $a^3 + b^3 + c^3 = 3abc$. Prove that $a + b + c = 0$. [5]

OR

If $\frac{x^2+1}{x} = 2\frac{1}{2}$, find the values of $x - \frac{1}{x}$ and $x^3 - \frac{1}{x^3}$.

33. A conical tent is 10 m high and the radius of its base is 24 m. Find the [5]
    a. slant height of the tent.
    b. cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

34. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (See the given figure). Show that [5]
    (i) $\Delta DAP \cong \Delta EBP$
    (ii) AD = BE

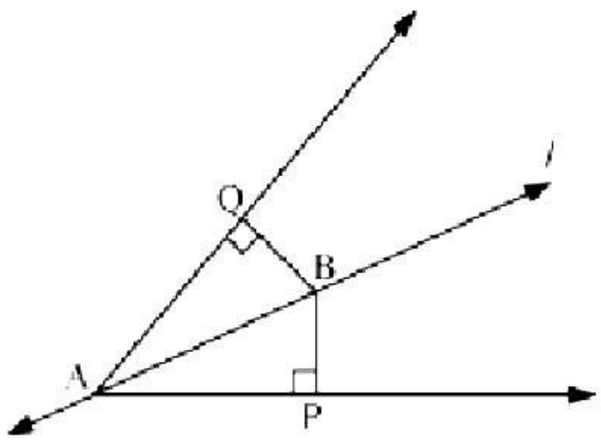


OR

Line *l* is the bisector of $\angle A$ and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see the given figure). Show that:

(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$.



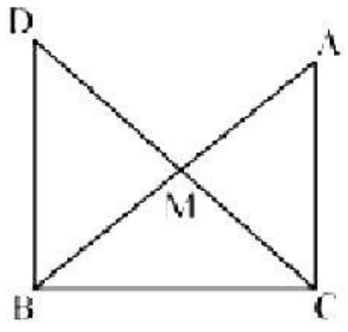
35. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that: [5]

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$



Section E

Case study-based questions are compulsory.

36. Decimal form of rational numbers can be classified into two types. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorization of q is of the form 2^n5^m, where n, m are non-negative and vice-versa.

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^n5^m, where n and m are non-negative integers. Then x has a non-terminating repeating decimal expansion.

Base on the above information, answer the following questions.

i. Check whether $\frac{81}{5^2 \times 2^3 \times 3}$ has terminating decimal expansion or not? [2]

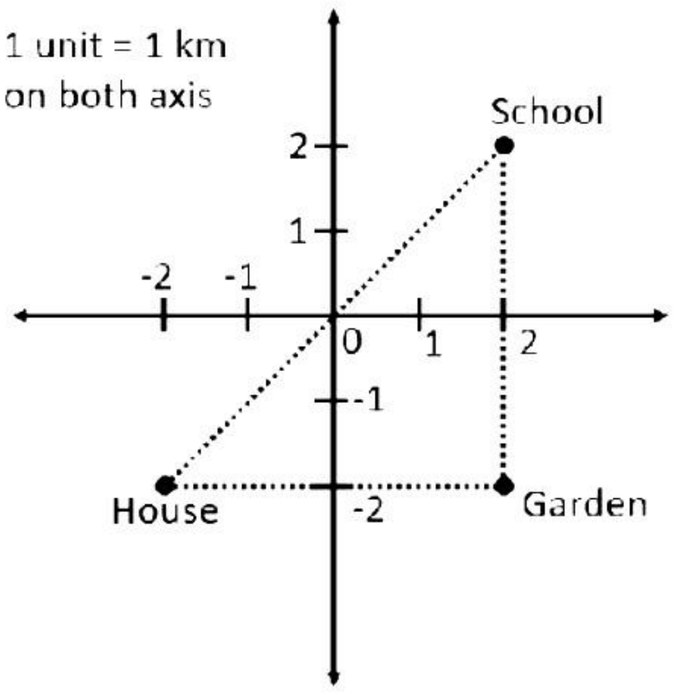
OR

Check whether $\frac{441}{2^2 \times 5^2 \times 7^2}$ has terminating decimal expansion or not? [2]

ii. For which value(s) of b, a rational number $\frac{251}{2^3 \times b^2}$ has a terminating decimal expansion? [1]

iii. Decimal expansion of a/an _____ number is non-terminating and non-repeating. [1]

37. Raj, studying in class 9, goes to his school every day by walking. There are two routes from his house to the school. One of the routes connects his house to the school directly, while the other takes a sharp left-turn around a garden. Both routes are shown in the graph below.



i. Find the distance from Raj's house to the garden. [1]

ii. Find the distance from garden to Raj's school. [1]

iii. Find the shortest distance from Raj's house to his school. [2]

OR

If Raj decides to travel by Auto rickshaw at the fare of Rs. 10/km, then how much will it cost to travel from his house to the garden? [2]

38. Meera bought some apples at the rate of Rs. 100 per kg and some avocados at the rate of Rs. 200 per kg. If she bought 'x' kg of apples and 'y' kg of avocado, then answer the following questions.

i. If the total cost incurred to Meera after buying 'x' kg of apples and 'y' kg of avocado is Rs. 1000, then write the equation representing this situation? [1]

ii. If Meera bought 3 kg of apples and 'y' kg of avocados for Rs. 500, find the value of 'y'. [2]

OR

If Meera bought 'x' kg of apples and 2 kg of avocados for Rs. 800, find the value of 'x'. [2]

iii. At what point the graph of $x + 2y = 10$ cuts the Y-axis? [1]

Solution

Section A

1. Correct option: A
   Solution:
   $3\sqrt{5} - 7\sqrt{3} - \left(5\sqrt{3} + 7\sqrt{5}\right)$
   $= 3\sqrt{5} - 7\sqrt{3} - 5\sqrt{3} - 7\sqrt{5}$
   $= -4\sqrt{5} - 12\sqrt{3}$
   $= -\left(4\sqrt{5} + 12\sqrt{3}\right)$

2. Correct option: C
   Solution:
   $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}}$
   $= \sqrt{\frac{\left(2-\sqrt{3}\right)^2}{2^2 - \left(\sqrt{3}\right)^2}}$
   $= \sqrt{\frac{\left(2-\sqrt{3}\right)^2}{4-3}}$
   $= 2 - \sqrt{3}$
   $= 2 - 1.7320$
   $= 0.268$

3. Correct option: B
   Solution:
   Given: r = 1.4 m and h = 6 m
   Capacity of the tank = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1.4)^2 \times 6 = 12.32 m^3$

4. Correct option: C
   Solution:
   Surface area of sphere = $4\pi r^2$
   Curved surface area of cylinder = $2\pi Rh$
   $\Rightarrow 4\pi r^2 = 2\pi Rh$
   $\Rightarrow r^2 = \frac{6 \times 12}{2}$
   $\Rightarrow r^2 = 36$
   $\Rightarrow r = 6$ cm

5. Correct option: C
   Solution:
   $p(x) = 6x^3 + x^2 + 2x - 1$
   $p(-1) = 6(-1)^3 + (-1)^2 + 2(-1) - 1$
   $= -6 + 1 - 2 - 1$
   $= -8$

6. Correct option: B
   Solution:
   Using identity: $(x + a)(x + b) = x^2 + (a + b)\,x + ab$
   Where $a = -5$ and $b = 9$
   $(x - 5)(x + 9) = x^2 + (-5 + 9)x + 9(-5)$
   $= x^2 + 4x - 45$

7. Correct option: A
   Solution:
   If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.
   $\therefore x = 3x - 20$
   $\therefore x - 3x = -20$
   $\therefore -2x = -20$
   $\therefore x = 10$

8. Correct option: C
   Solution:
   The sum of the angles of a triangle is 180°.
   $\therefore x^\circ + 3x^\circ + 5x^\circ = 180^\circ$
   $\therefore 9x = 180^\circ$
   $\therefore x = 20^\circ$

9. Correct option: B
   Solution:
   In triangle PQR, $\angle P : \angle Q : \angle R = 3 : 2 : 1$
   Let the angles of the triangle be 3x, 2x and x.
   $\angle PRT = 90^\circ$
   In triangle PQR,
   $\angle P + \angle Q + \angle R = 180^\circ$
   $\because$ Sum of the angles in a triangle = 180°
   $\therefore 3x + 2x + x = 180^\circ$
   $\therefore 6x = 180^\circ$
   $\therefore x = 30^\circ$
   $\therefore \angle PRQ = x = 30^\circ$
   $\angle PRQ + \angle PRT + \angle TRS = 180^\circ$ .... (Linear pair)
   $\therefore 30^\circ + 90^\circ + \angle TRS = 180^\circ$
   $\therefore \angle TRS = 60^\circ$

10. Correct option: A
    Solution:
    From the given information,
    QR < PR < PQ
    ∴ ∠P < ∠Q < ∠R ... (Opposite angles of sides)
    Hence, ∠P is the least angle of the given triangle.

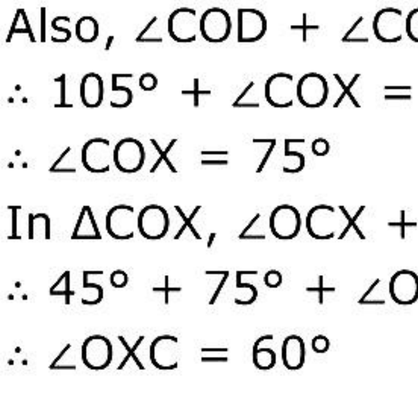
11. Correct option: C
    Solution:
    In the parallelogram, opposite angles are congruent.
    ∴ ∠A = ∠C = 70°

12. Correct option: B
    Solution:
    Let the measure of the equal angles of the given quadrilateral be x.
    The measure of the other two angles is 60° each.
    ∴ x + x + 60° + 60° = 360°
    ∴ 2x = 240°
    ∴ x = 120°

13. Correct option: B
    Solution:
    The angles of a square are bisected by the diagonals.
    ∠OCX = 45°
    Also, ∠COD + ∠COX = 180° ... (Linear pair of angles)
    ∴ 105° + ∠COX = 180°
    ∴ ∠COX = 75°
    In ΔCOX, ∠OCX + ∠COX + ∠OXC = 180°
    ∴ 45° + 75° + ∠OXC = 180°
    ∴ ∠OXC = 60°

14. Correct option: C
    Solution:
    If a transversal line intersects two parallel lines, then the corresponding angles and alternate angles are equal, while the interior angles are supplementary.

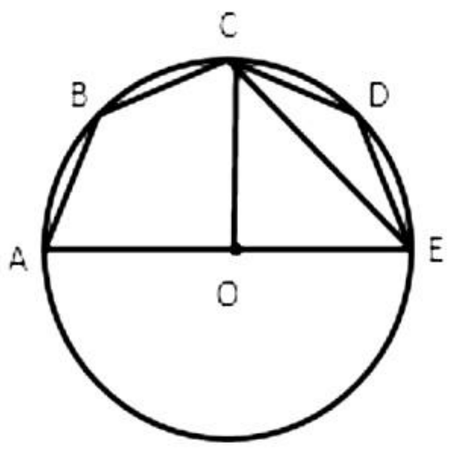
15. Correct option: D
    Solution:



    Join OB.

In ΔAOB,
AO = BO ∵ Radii
Opposite angles of equal sides
∴ ∠OBA = ∠OAB = 30° .....(i)
Similarly, in ΔBOC,
OB = OC
∴ ∠OBC = ∠OCB = 40° .....(ii)
∠ABO + ∠OBC = ∠ABC
∴ ∠ABC = 30° + 40° from (i) and (ii)
∴ ∠ABC = 70° ∵ angle subtended by an arc
∠ABC = ½ ∠AOC
∴ 70° = ½ ∠AOC
∴ ∠AOC = 140°

16. Correct option: A
Solution:
Join centre OC and EC.



∠AOC = 180°/2 = 90° and ∠AOC = 2∠AEC
Angle at the centre is twice the angle at the circumference subtended by the same chord.
∴ ∠AEC = 90°/2 = 45°
Now, ABCE is a cyclic quadrilateral.
∴ ∠ABC + ∠AEC = 180°
Pair of opposite angles in a cyclic quadrilateral are supplementary.
∴ ∠ABC + 45° = 180°
∴ ∠ABC = 135°
Similarly, ∠CDE = 135°
∴ ∠ABC + ∠CDE = 135° + 135° = 270°

17. Correct option: B
Solution:
x + 110° = 180°, as they are interior angles on the same side of the transversal.
x = 70°

18. Correct option: D
Solution:
Points, lines and planes are undefined terms in geometry.

19. Correct option: C

Solution:

As ABCD is a cyclic quadrilateral, we have

$\angle A + \angle C = 180°$ ... (Opposite angles of a cyclic quadrilateral are supplementary)

$\therefore 120° + \angle C = 180°$

$\therefore \angle C = 60°$

So, the assertion is true.

Since, opposite angles of a cyclic quadrilateral are supplementary.

Thus, the reason is false.

20. Correct option: A

Solution:

Let the angle be x.

As the sum of two complementary angles is 90°.

According to the question, we have

$x = 2(90° - x)$

$\therefore x = 180° - 2x$

$\therefore 3x = 180°$

$\therefore x = 60°$

$\therefore$ The measure of the angle is 60°.

Thus, the assertion is true.

Also, the reason is true and it is the correct explanation of assertion.

Section B

21. $p(x) = x^4 + x^2 - 8x + 6$

$p(1) = 1 + 1 - 8 + 6$

$p(1) = 0$

Hence, $(x - 1)$ is the factor of the given polynomial or $x = 1$ is the zero of the given polynomial.

22. $203 \times 205 = (200 + 3)(200 + 5)$

$= 200^2 + (3 + 5) \times 200 + 3 \times 5$ $\quad \because (x + a)(x + b) = x^2 + (a + b)x + ab$

$= 40000 + 1600 + 15$

$= 41615$

23.

$$
\begin{array}{r}
1.428571 \\
7\overline{)\,10} \\
\underline{-07} \\
30 \\
\underline{-28} \\
20 \\
\underline{-14} \\
60 \\
\underline{-56} \\
40 \\
\underline{-35} \\
50 \\
\underline{-\,49} \\
10 \\
\underline{-\ \ 7} \\
3
\end{array}
$$

$\frac{10}{7} = 1.\overline{428571}$

24.

$\frac{13}{20} = \frac{13 \times 5}{20 \times 5} = \frac{65}{100} = 0.65$

Therefore, the given number is a terminating decimal.

OR

Let x = 0.7777... (i)

10x = 7.7777... (ii)

Subtracting (i) from (ii),

9x = 7

$x = \frac{7}{9}$

25.

$\left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{-5}{4}\right)^3$

Consider $x = \frac{1}{2}$, $y = \frac{3}{4}$ and $z = \frac{-5}{4}$

$\therefore x + y + z = \frac{1}{2} + \frac{3}{4} + \left(\frac{-5}{4}\right) = \frac{5}{4} - \frac{5}{4} = 0$

$\therefore x^3 + y^3 + z^3 = 3xyz$

$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{-5}{4}\right)^3 = 3 \times \frac{1}{2} \times \frac{3}{4} \times \frac{-5}{4} = \frac{-45}{32}$

OR

$$\left(\frac{a}{2}-\frac{b}{3}\right)^3=\left(\frac{a}{2}\right)^3-\left(\frac{b}{3}\right)^3-3\times\frac{a}{2}\times\frac{b}{3}\left(\frac{a}{2}-\frac{b}{3}\right)$$

$$=\frac{a^3}{8}-\frac{b^3}{27}-\frac{ab}{2}\left(\frac{a}{2}-\frac{b}{3}\right)$$

$$=\frac{a^3}{8}-\frac{b^3}{27}-\frac{a^2b}{4}+\frac{ab^2}{6}$$

Section C

26. Let the common ratio between the sides of given triangle be x.

So, the sides of a triangle will be 12x, 17x, and 25x, respectively.

Perimeter of this triangle = 540 cm

Then, 12x + 17x + 25x = 540 cm

$\Rightarrow$ 54x = 540 cm

$\Rightarrow$ x = 10 cm

Hence, the sides of a triangle will be 120 cm, 170 cm, and 250 cm.

$$\text{Semi-perimeter of a triangle} = \frac{\text{perimeter of triangle}}{2} = \frac{540 \text{ cm}}{2} = 270 \text{ cm}$$

By Heron's formula

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{270(270-120)(270-170)(270-250)}\right] \text{cm}^2$$
$$= \left[\sqrt{270 \times 150 \times 100 \times 20}\right] \text{cm}^2$$
$$= 9000 \text{ cm}^2$$

So, the area of this triangle will be 9000 cm^2.

27. $(2x - 3)$ is a factor of $p(x) = 2x^3 - 9x^2 + x + k$.

Then $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

That is, if $2x - 3$ is a factor of $p(x)$, then

$$p\left(\frac{3}{2}\right) = 0$$

$$p(x) = 2x^3 - 9x^2 + x + k$$

$$\Rightarrow 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + k = 0$$

$$\Rightarrow 2 \times \frac{27}{8} - 9 \times \frac{9}{4} + \frac{3}{2} + k = 0$$

$$\Rightarrow \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + k = 0$$

$$\Rightarrow \frac{27 - 81 + 6}{4} + k = 0$$

$$\Rightarrow -12 + k = 0$$

$$\Rightarrow k = 12$$

28.

(i) In ΔAPB and ΔCQD,

$\angle APB = \angle CQD$	(each 90°)
$AB = CD$	(opposite sides of a parallelogram ABCD)
$\angle ABP = \angle CDQ$	(alternate interior angles for AB || CD)
$\therefore \Delta APB \cong \Delta CQD$	(by AAS congruency)

(ii) By using the result obtained as above,

$\Delta APB \cong \Delta CQD$

$\Rightarrow AP = CQ$ (by CPCT)

OR

(i) In ΔADC, S and R are the mid-points of sides AD and CD, respectively.
In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ ... (1)

(ii) In ΔABC, P and Q are the mid-points of sides AB and BC, respectively. So, by using mid-point theorem, we have

PQ || AC and PQ = $\frac{1}{2}$AC ... (2)

Now using equations (1) and (2), we have

PQ || SR and PQ = SR ... (3)

$\Rightarrow$ PQ = SR

(iii) From equation (3), we have

PQ || SR and PQ = SR

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

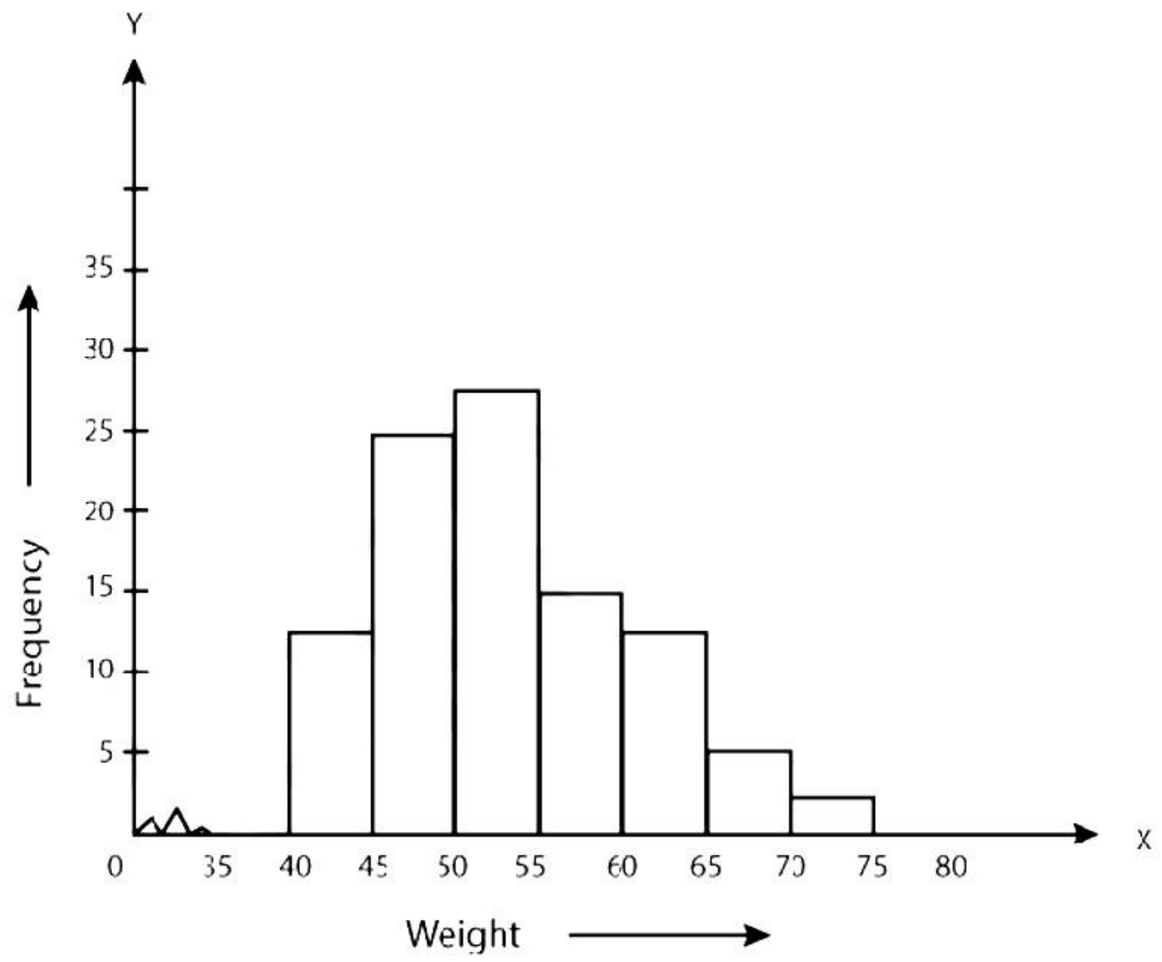
Hence, PQRS is a parallelogram.

29. Steps of construction:

i. We represent the weights on the horizontal axis by choosing the scale as 1 cm = 5 kg. Also, since the first class interval is starting from 35 and not zero, we show it on the graph by marking a kink or a break on the axis.

ii. We represent the number of people (frequency) on the vertical axis. Since the maximum frequency is 28, we choose the scale as 1 cm = 5 people.

iii. We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals. The histogram is as follows:



30. Slant height (l) of a conical tomb = 25 m

Base radius (r) of a tomb = $\frac{14}{2}$m = 7 m

CSA of a conical tomb = $\pi rl = \left(\frac{22}{7} \times 7 \times 25\right)$ m^2 = 550 m^2

Cost of white-washing 100 m^2 area = Rs. 210

Then, cost of white-washing 550 m^2 area = Rs. $\left(\frac{210 \times 550}{100}\right)$ = Rs. 1155

Thus, the cost of white-washing the conical tomb is Rs. 1155.

OR

Radius (r) of a cone = $\frac{40}{2}$ cm = 20 cm = 0.2 m

Height (h) of a cone = 1 m

Slant height (l) of cone = $\sqrt{h^2 + r^2} = \left[\sqrt{(1)^2 + (0.2)^2}\right]$ m = $\left(\sqrt{1.04}\right)$ m = 1.02 m

CSA of each cone = πrl = (3.14 × 0.2 × 1.02) m^2 = 0.64056 m^2

CSA of 50 such cones = (50 × 0.64056) m^2 = 32.028 m^2

Cost of painting 1 m^2 area = Rs. 12

⇒ Cost of painting 32.028 m^2 area = Rs. (32.028 × 12) = Rs. 384.336 ≈ Rs. 384

Thus, it will cost approximately Rs. 384 for painting 50 hollow cones.

31.

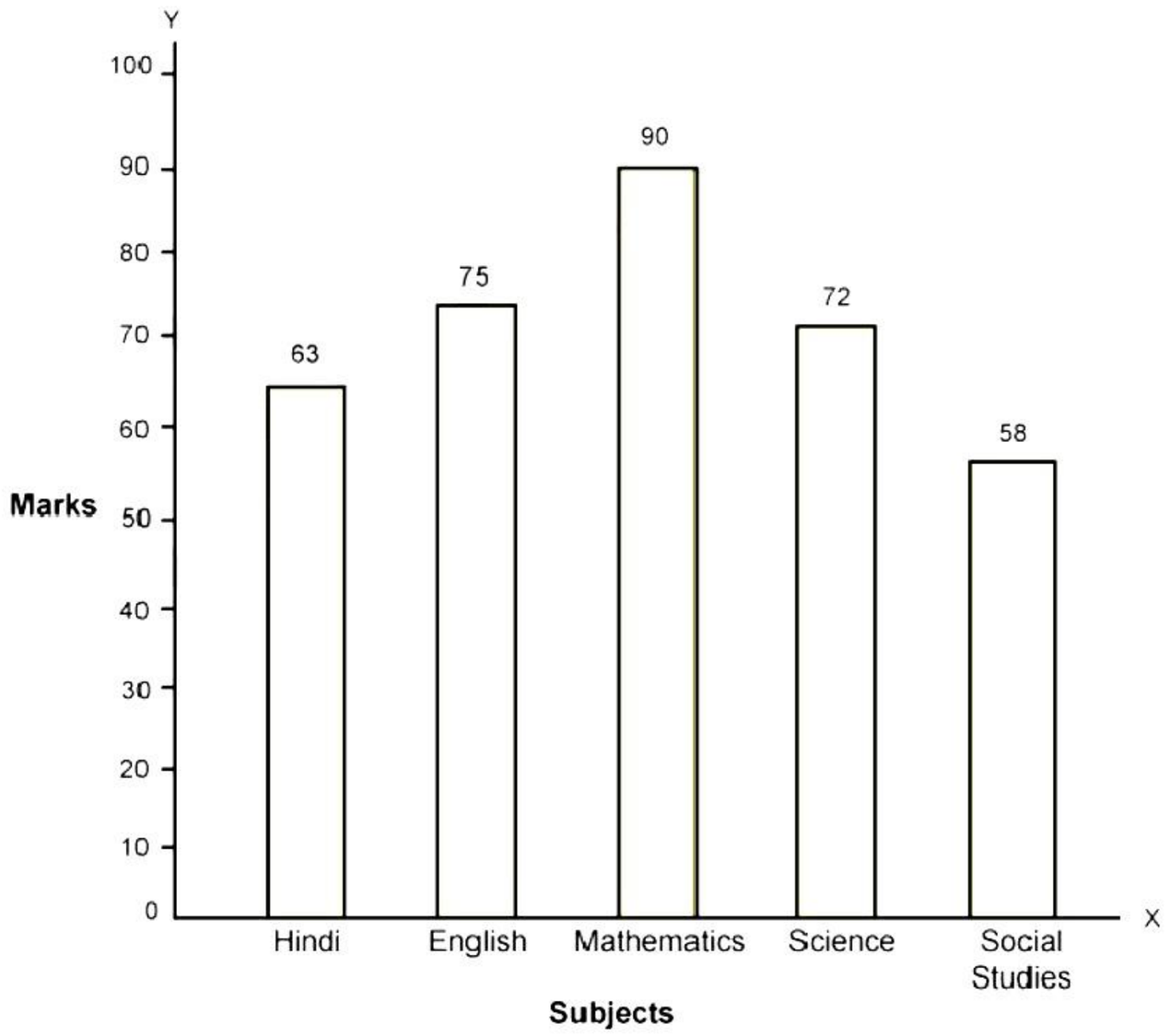
A bar graph can be drawn using the following steps:

i. On a graph paper, draw a horizontal line OX and vertical line OY, representing the X-axis and Y-axis, respectively.

ii. Along OX, write the names of the subjects at points taken at uniform gaps. Along OY, write the marks with the scale as 1 unit = 10 marks.

iii. Heights of the various bars are Hindi – 63; English – 75; Mathematics – 90; Science – 72; Social Science – 58.

iv. On the x-axis, draw bars of equal width and height obtained in step (iii).



Section D

32. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$$= \frac{1}{2}(a+b+c)\left(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right)$$

$$= \frac{1}{2}(a+b+c)\left(a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2\right)$$

$$= \frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

Given that $a^3 + b^3 + c^3 - 3abc = 0$

$$(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0 \qquad \ldots.(i)$$

Since $a - b \neq 0$, we notice that $(a - b)^2 > 0$.

Similarly, $(b - c)^2 > 0$ and $(c - a)^2 > 0$.

Hence, $(a - b)^2 + (b - c)^2 + (c - a)^2 > 0$.

Now in (i), we have the product of two numbers is equal to 0 and one of them is positive.

Hence, the other number must be equal to 0.

So, we conclude that $a + b + c = 0$.

OR

$$\frac{x^2+1}{x} = 2\frac{1}{2}$$

$$\Rightarrow x + \frac{1}{x} = \frac{5}{2}$$

Now,

$$\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$$

$$= \left(\frac{5}{2}\right)^2 - 4$$

$$= \frac{25}{4} - 4$$

$$= \frac{9}{4}$$

$$\therefore x - \frac{1}{x} = \pm\frac{3}{2}$$

Case I: $x - \frac{1}{x} = \frac{3}{2}$

$$\left(x-\frac{1}{x}\right)^3=\left(\frac{3}{2}\right)^3$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=\frac{27}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\times\frac{3}{2}=\frac{27}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{27}{8}+\frac{9}{2}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{63}{8}=7\frac{7}{8}$$

Case II: $x-\frac{1}{x}=-\frac{3}{2}$

$$\left(x-\frac{1}{x}\right)^3=\left(-\frac{3}{2}\right)^3$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=-\frac{27}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\times\left(-\frac{3}{2}\right)=-\frac{27}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}=-\frac{27}{8}-\frac{9}{2}$$

$$\Rightarrow x^3-\frac{1}{x^3}=-\frac{63}{8}=-7\frac{7}{8}$$

$$\therefore x^3-\frac{1}{x^3}=\pm7\frac{7}{8}$$

33.

(i) Height (h) of conical tent = 10 m
Radius (r) of conical tent = 24 m
Let the slant height of conical tent = l
$l^2 = h^2 + r^2 = (10\text{ m})^2 + (24\text{ m})^2 = 676\text{ m}^2$
$\therefore$ l = 26 m
Thus, the slant height of the conical tent is 26 m.

(ii) CSA of a tent = $\pi rl = \left(\frac{22}{7}\times24\times26\right)\text{ m}^2 = \frac{13728}{7}\text{m}^2$

Cost of 1 m^2 canvas = Rs. 70

Then, cost of $\frac{13728}{7}m^2$ canvas = Rs. $\left(\frac{13728}{7}\times70\right)$ = Rs. 137280

Thus, the cost of canvas required to make the tent is Rs. 137280.

34. Given that $\angle EPA = \angle DPB$

$\Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\Rightarrow \angle DPA = \angle EPB$ ....(1)

Now, in ΔDAP and ΔEBP

$\angle DAP = \angle EBP$ (given)

$AP = BP$ (P is the mid-point of AB)

$\angle DPA = \angle EPB$ [From (1)]

$\therefore \Delta DAP \cong \Delta EBP$ (ASA congruence rule)

$\therefore AD = BE$ (by CPCT)

OR

In ΔAPB and ΔAQB,

$\angle APB = \angle AQB$ (each 90^o)

$\angle PAB = \angle QAB$ (l is angle bisector of $\angle A$)

$AB = AB$ (common)

$\therefore \Delta APB \cong \Delta AQB$ (by AAS congruence rule)

$\therefore BP = BQ$ (by CPCT)

Or we can say that B is equidistant from the arms of $\angle A$.

35.

(i) In ΔAMC and ΔBMD,

$AM = BM$ (M is the mid-point of AB)

$\angle AMC = \angle BMD$ (vertically opposite angles)

$CM = DM$ (given)

$\therefore \Delta AMC \cong \Delta BMD$ (by SAS congruence rule)

(ii) $\angle ACM = \angle BDM$ (by CPCT as $\Delta AMC \cong \Delta BMD$)

But $\angle ACM$ and $\angle BDM$ are alternate interior angles.

Since alternate angles are equal,

$DB \parallel AC$

$\Rightarrow \angle DBC + \angle ACB = 180^o$ (co-interior angles)

$\Rightarrow \angle DBC + 90^o = 180^o$

$\Rightarrow \angle DBC = 90^o$

(iii) In ΔDBC and ΔACB,

$DB = AC$ (by CPCT as $\Delta AMC \cong \Delta BMD$)

$\angle DBC = \angle ACB$ (each 90°)

$BC = CB$ (Common)

$\therefore \Delta DBC \cong \Delta ACB$ (SAS congruence rule)

(iv) We have $\Delta DBC \cong \Delta ACB$

$\therefore CD = AB$ (by CPCT)

$\Rightarrow 2CM = AB$

$\therefore CM = \frac{1}{2} AB$

Section E

Case study-based questions are compulsory.

36.

i. Given, $\frac{81}{5^2 \times 2^3 \times 3}$

Now, $\frac{81}{5^2 \times 2^3 \times 3} = \frac{27 \times 3}{5^2 \times 2^3 \times 3} = \frac{27}{5^2 \times 2^3} = \frac{27}{200}$

It is of the form $\frac{p}{q}$, where p and q are co-prime

We observe that prime factorixation of q is of the form 2^n5^m.
Hence, it has a terminating decimal expansion.

That is, $\frac{81}{5^2 \times 2^3 \times 3} = 0.135$

OR

Given, $\frac{441}{2^2 \times 5^2 \times 7^2}$

Now, $\frac{441}{2^2 \times 5^2 \times 7^2} = \frac{9 \times 7 \times 7}{2^2 \times 5^2 \times 7^2} = \frac{9}{2^2 \times 5^2}$

It is of the form $\frac{p}{q}$.

We observe that prime factorixation of q is of the form 2^n5^m.
Hence, it has a terminating decimal expansion.

That is, $\frac{441}{2^2 \times 5^2 \times 7^2} = 0.09$

ii. When rational number is of the form $\frac{p}{q}$, where p, q are coprime, it has terminating decimal expansion if q is of the form 2^n5^m.

In $\frac{251}{2^3 \times b^2}$, $q = 2^3 \times b^2 \Rightarrow b = 5$.

Hence, for b = 5, $\frac{251}{2^3 \times b^2}$ will be a terminating decimal.

iii. Decimal expansion of an irrational number is non-terminating and non-repeating.

37.

i. The distance from Raj's house to the garden is 4 km.

ii. The distance from garden to Raj's school is 4 km.

iii. By applying Pythagoras theorem, we get the shortest distance from Raj's house to his school = $\sqrt{4^2 + 4^2} = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$ km

OR

Distance from Raj's house to the garden = 4 km

Auto rickshaw fare = Rs. 10/km

Hence, travelling cost from his house to garden = Rs. (4 × 10) = Rs. 40

38.

i. Cost of 'x' kg of apples = Rs. 100x

Cost of 'y' kg of avocados = Rs. 200y

Total cost = Rs. 1000

⇒ Rs. (100x + 200y) = Rs. 1000

⇒ 100x + 200y = 1000

⇒ x + 2y = 10

ii. Cost of 3 kg of apples = Rs. (3 × 100) = Rs. 300

Cost of 'y' kg of avocados = Rs. 200y

Total cost = Rs. 500

⇒ 300 + 200y = 500

⇒ 200y = 200

⇒ y = 1

OR

Cost of 'x' kg of apples = Rs. 100x

Cost of 2 kg of avocados = Rs. (2 × 200) = Rs. 400

Total cost = Rs. 800

⇒ 100x + 400 = 800

⇒ 100x = 400

⇒ x = 4

iii. x + 2y = 10

Substituting x = 0 in x + 2y = 10, we get

0 + 2y = 10

⇒ y = 5

So, the graph of x + 2y = 10 cuts the Y-axis at point (0, 5).